

SmallClassNr

Library of finite groups with small class number

1.4.3

13 November 2025

Sam Tertooy

Sam Tertooy

Email: sam.tertooy@kuleuven.be

Homepage: <https://stertooy.github.io/>

Address: Wiskunde

KU Leuven, Kulak Kortrijk Campus

Etienne Sabbelaan 53

8500 Kortrijk

Belgium

Abstract

The SmallClassNr package provides access to finite groups with small class number. Currently, it contains the finite groups of class number at most 14.

Copyright

© 2022–2025 Sam Tertooy

The SmallClassNr package is free software, it may be redistributed and/or modified under the terms and conditions of the [GNU Public License Version 2](#) or (at your option) any later version.

Acknowledgements

This documentation was created using the GAPDoc and AutoDoc packages.

Contents

1	The SmallClassNr package	4
1.1	Installation	4
1.2	Loading	4
1.3	Citing	5
1.4	Support	5
2	Classification	6
3	The Small Class Number Library	7
3.1	Functions	7
	References	10
	Index	11

Chapter 1

The SmallClassNr package

This is the manual for the GAP 4 package SmallClassNr version 1.4.3, developed by Sam Tertoooy.

1.1 Installation

If you are using GAP version 4.15.0 or newer, then SmallClassNr should be installed by default.

If this is not the case, but the `PackageManager` package is installed and loaded, you can install SmallClassNr from within a GAP session using `InstallPackage` (**PackageManager: InstallPackage**).

Example

```
gap> InstallPackage( "SmallClassNr" );  
...  
true
```

Alternatively, you can download SmallClassNr as a .tar.gz archive [here](#). After extracting, you should place it in a suitable pkg folder. For example, on a Debian-based Linux distribution (e.g. Ubuntu, Mint), you can place it in `$HOME/.gap/pkg` (recommended) which makes it available for just yourself, or in the GAP installation directory (`gap-X.Y.Z/pkg`) which makes it available for all users.

You can use the following command to efficiently install the package for yourself:

Command

```
wget -q0 - https://[...].tar.gz | tar xzf - --one-top-level=$HOME/.gap/pkg
```

1.2 Loading

Once installed, loading SmallClassNr can be done by using `LoadPackage` (**Reference: LoadPackage**).

Example

```
gap> LoadPackage( "SmallClassNr" );  
...  
true
```

1.3 Citing

If you use the `SmallClassNr` package in your research, we would love to hear about your work via an email to the address sam.tertooy@kuleuven.be. If you have used the `SmallClassNr` package in the preparation of a paper and wish to refer to it, please cite it as described below.

In Bib_T_EX:

BibTeX

```
@misc{SCN1.4.3,
  author = {Tertooy, Sam},
  title = {{SmallClassNr,
    Library of finite groups with small class number,
    Version 1.4.3}},
  note = {GAP package},
  year = {2025},
  howpublished = {\url{https://stertooy.github.io/SmallClassNr}}
}
```

In Bib_L_A_T_EX:

BibLaTeX

```
@software{SCN1.4.3,
  author = {Tertooy, Sam},
  title = {SmallClassNr},
  subtitle = {Library of finite groups with small class number},
  version = {1.4.3},
  note = {GAP package},
  year = {2025},
  url = {https://stertooy.github.io/SmallClassNr}
}
```

1.4 Support

If you encounter any problems, please submit them to the [issue tracker](#). If you have any questions on the usage or functionality of `SmallClassNr`, you may contact me via email at sam.tertooy@kuleuven.be.

Chapter 2

Classification

The *class number* $k(G)$ of a group G is the number of conjugacy classes of G . In 1903, Landau proved in [Lan03] that for every $n \in \mathbb{N}$, there are only finitely many finite groups with exactly n conjugacy classes. The `SmallClassNr` package provides access to the finite groups with class number at most 14.

These groups were classified in the following papers:

- $k(G) \leq 5$, by Miller in [Mil11] and independently by Burnside in [Bur11]
- $k(G) = 6, 7$, by Poland in [Pol68]
- $k(G) = 8$, by Kosvintsev in [Kos74]
- $k(G) = 9$, by Odincov and Starostin in [OS76]
- $k(G) = 10, 11$, by Vera López and Vera López in [VLVL85] (1)
- $k(G) = 12$, by Vera López and Vera López in [VLVL86] (2)
- $k(G) = 13, 14$, by Vera López and Sangroniz in [VLS07]

(1) In [VLVL85], three distinct groups of the form $(C_5 \times C_5) \rtimes C_4$ order 100 with class number 10 are given. However, only two such groups exist, being the ones with `IdClassNr` equal to `[10, 25]` and `[10, 26]`.

(2) In [VLVL86], only 48 groups with class number 12 are listed. The three missing groups are provided in the appendix of [VLS07]. These are the groups with `IdClassNr` equal to `[12, 13]`, `[12, 16]` and `[12, 39]`.

Chapter 3

The Small Class Number Library

3.1 Functions

3.1.1 SmallClassNrGroup

▷ `SmallClassNrGroup(k, i)` (function)

Returns: the i -th finite group of class number k in the library.

Alternatively, the pair $[k, i]$ can be given as a single argument id . If the group is solvable, it is given as a `PcGroup` whose `Pcgs` is a `SpecialPcgs`. If the group is not solvable, it will be given as a permutation group of minimal permutation degree and with a minimal generating set.

Example

```
gap> G := SmallClassNrGroup( 6, 4 );
<pc group of size 18 with 3 generators>
gap> NrConjugacyClasses( G );
6
gap> IsDihedralGroup( G );
true
```

3.1.2 SmallClassNrGroupsAvailable

▷ `SmallClassNrGroupsAvailable(k)` (function)

Returns: true if the finite groups of class number k are available in the library, and false otherwise.

Example

```
gap> SmallClassNrGroupsAvailable( 14 );
true
gap> SmallClassNrGroupsAvailable( 15 );
false
```

3.1.3 AllSmallClassNrGroups

▷ `AllSmallClassNrGroups(arg)` (function)

Returns: all finite groups with certain properties as specified by arg .

The arguments must come in pairs consisting of a function and a value (or list of possible values). At least one of the functions must be `NrConjugacyClasses`. Missing functions will be interpreted as `NrConjugacyClasses`, missing values as `true`.

Example

```

gap> L1 := AllSmallClassNrGroups( [3..5], IsNilpotent );
[ <pc group of size 3 with 1 generator>,
  <pc group of size 4 with 2 generators>,
  <pc group of size 4 with 2 generators>,
  <pc group of size 5 with 1 generator>,
  <pc group of size 8 with 3 generators>,
  <pc group of size 8 with 3 generators> ]
gap> List( L1, NrConjugacyClasses );
[ 3, 4, 4, 5, 5, 5 ]
gap> L2 := AllSmallClassNrGroups( IsSolvable, true, NrConjugacyClasses, 6 );
[ <pc group of size 6 with 2 generators>,
  <pc group of size 12 with 3 generators>,
  <pc group of size 12 with 3 generators>,
  <pc group of size 18 with 3 generators>,
  <pc group of size 18 with 3 generators>,
  <pc group of size 36 with 4 generators>,
  <pc group of size 72 with 5 generators> ]
gap> ForAll( L2, G -> IsSolvable( G ) and NrConjugacyClasses( G ) = 6 );
true

```

3.1.4 OneSmallClassNrGroup

▷ OneSmallClassNrGroup(arg)

(function)

Returns: one finite group with certain properties as specified by *arg*.

The arguments must come in pairs consisting of a function and a value (or list of possible values). At least one of the functions must be `NrConjugacyClasses`. Missing functions will be interpreted as `NrConjugacyClasses`, missing values as `true`.

Example

```

gap> H := OneSmallClassNrGroup( 6, IsAbelian );
<pc group of size 6 with 2 generators>
gap> IsCyclic( H );
true
gap> K := OneSmallClassNrGroup( 10, IsSolvable, true, IsNilpotent, false );
<pc group of size 28 with 3 generators>
gap> NrConjugacyClasses( K ) = 10 and IsSolvable( K ) and not IsNilpotent( K );
true

```

3.1.5 NrSmallClassNrGroups

▷ NrSmallClassNrGroups(arg)

(function)

Returns: the number of finite groups with certain properties as specified by *arg*.

The arguments must come in pairs consisting of a function and a value (or list of possible values). At least one of the functions must be `NrConjugacyClasses`. Missing functions will be interpreted as `NrConjugacyClasses`, missing values as `true`.

Example

```

gap> NrSmallClassNrGroups( 14 );
93
gap> NrSmallClassNrGroups( [3..5], IsNilpotentGroup );
6

```



```
gap> NrSmallClassNrGroups( IsSolvable, true, NrConjugacyClasses, 6 );
7
```

3.1.6 IteratorSmallClassNrGroups

▷ IteratorSmallClassNrGroups(*arg*) (function)

Returns: an iterator that iterates over the finite groups with properties as specified by *arg*. The arguments must come in pairs consisting of a function and a value (or list of possible values). At least one of the functions must be NrConjugacyClasses. Missing functions will be interpreted as NrConjugacyClasses, missing values as true.

Example

```
gap> iter := IteratorSmallClassNrGroups( IsSolvable, false, 11 );
<iterator>
gap> for G in iter do Print( Size( G ), "\n" ); od;
336
720
720
1344
1344
1512
2448
29120
```

3.1.7 IdClassNr

▷ IdClassNr(*G*) (attribute)

Returns: the SmallClassNr ID of *G*, i.e. a pair [*k*, *i*] such that *G* is isomorphic to SmallClassNrGroup(*k*, *i*).

Example

```
gap> IdClassNr( AlternatingGroup( 5 ) );
[ 5, 8 ]
gap> A := SmallClassNrGroup( 5, 8 );
Group([ (1,2,3), (1,4,5) ])
gap> IsAlternatingGroup( A );
true
```

References

- [Bur11] W. Burnside. *Theory of groups of finite order*, chapter Note A, pages 461–462. The University Press, second edition, 1911. [6](#)
- [Kos74] L. F. Kosvintsev. Over the theory of groups with properties given over the centralizers of involutions. *Sverdlovsk (Ural.), Summary thesis Doct*, 1974. [6](#)
- [Lan03] E. Landau. Über die Klassenzahl der binären quadratischen Formen von negativer Discriminante. *Math. Ann.*, 56(4):671–676, 1903. [6](#)
- [Mil11] G. A. Miller. Groups involving only a small number of sets of conjugate operators. *Arch. der Math. u. Phys.*, 17:199–204, 1911. [6](#)
- [OS76] V. A. Odincov and A. I. Starostin. Finite groups with 9 classes of conjugate elements. *Ural. Gos. Univ. Mat. Zap.*, 10:114–134, 1976. [6](#)
- [Pol68] J. Poland. Finite Groups with a given Number of Conjugate Classes. *Canadian J. Math.*, 20:456–464, 1968. [6](#)
- [VLS07] A. Vera López and J. Sangroniz. The finite groups with thirteen and fourteen conjugacy classes. *Math. Nachr.*, 280(5-6):676–694, 2007. [6](#)
- [VLVL85] A. Vera López and J. Vera López. Classification of finite groups according to the number of conjugacy classes. *Isr. J. Math.*, 51(4):305–338, 1985. [6](#)
- [VLVL86] A. Vera López and J. Vera López. Classification of finite groups according to the number of conjugacy classes II. *Isr. J. Math.*, 56(2):188–221, 1986. [6](#)

Index

AllSmallClassNrGroups, [7](#)

IdClassNr, [9](#)

IteratorSmallClassNrGroups, [9](#)

NrSmallClassNrGroups, [8](#)

OneSmallClassNrGroup, [8](#)

SmallClassNrGroup, [7](#)

SmallClassNrGroupsAvailable, [7](#)